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Superparticles in $D > 11$

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ABSTRACT

Actions for two-superparticle system in $(10, 2)$ dimensions and three-superparticle systems in $(11, 3)$ dimensions are constructed. These actions have worldline bosonic and fermionic local symmetries, and target space global supersymmety generalizing the reparametrization, κ-symmetry and Poincaré supersymmetry of the usual superparticle. With the second particle, or the second and third particles on-shell, they describe a superparticle propagating in the background of a second superparticle in $(10, 2)$ dimensions, or two other superparticles in $(11, 3)$ dimensions. Symmetries of the action are shown to exist in presence of super Yang-Mills background as well.

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1 Introduction

The possibility of a super p-brane in $(10,2)$-dimensions was conjectured long ago [1], in the context of a generalized brane-scan. More recently, there have been indications for the existence of a (10,2) dimensional structure in M-theory [2, 3, 4]. Motivated by these considerations, super Yang-Mills equations of motion in (10,2) dimensions were constructed in [5]. This result has been recently generalized to describe the equations of motion of supergravity in (10,2) dimensions [6]. Previously, possible existence of hidden symmetries descending from (11,2) dimensions was pointed out [7]. Recently [8], it has been suggested that there may be a $(11,3)$ dimensional structure in the master theory, and even the possibility of a $(12,4)$ dimensional structure has been speculated in [9]. These considerations motivated one of the authors to look for an extension of the work presented in [5] to higher dimensions, and it was found that the construction of [5] generalizes naturally to $(11,3)$ dimensions. An extension beyond $(11,3)$ dimensions ran into an obstacle [10], which has been removed in [11], where super Yang- Mills equations have been constructed in $(8+n,n)$ dimensions, for any $n \geq 1$.

The symmetry algebras realized in the field theoretic models just mentioned are [7, 5, 10, 12]:

$$
\begin{aligned}
(10,2): \qquad \{Q_\alpha, Q_\beta\} &= (\gamma^{\mu\nu})_{\alpha\beta}\, P_\mu\, n_\nu \,, &(1)\\
(11,3): \qquad \{Q_\alpha, Q_\beta\} &= (\gamma^{\mu\nu\rho})_{\alpha\beta}\, P_\mu\, n_\nu\, m_\rho \,, &(2)
\end{aligned}
$$

where n_μ and m_μ are mutually orthogonal constant null vectors. These break the $(10,2)$ or $(11,3)$ dimensional covariance. In order to maintain this covariance, it is natural to replace the null vectors by momentum generators [7] (see also [8]), thereby obtaining [2]

$$
\begin{aligned}
(10,2): \qquad \{Q_\alpha, Q_\beta\} &= (\gamma^{\mu\nu})_{\alpha\beta}\, P_{1\mu}\, P_{2\nu} \,, &(3)\\
(11,3): \qquad \{Q_\alpha, Q_\beta\} &= (\gamma^{\mu\nu\rho})_{\alpha\beta}\, P_{1\mu}\, P_{2\nu}\, P_{3\rho} \,. &(4)
\end{aligned}
$$

The algebras (1) and (3) first made their appearances in [7], and (2) and (4) in [10, 12]. In particular, (3) has been put to use in [4, 7] in the context of higher dimensional unification of duality symmetries; in [13] where four dimensional bi-local field theoretic realizations are given and interesting physical consequences such as family unification are suggested; and in [8], where a two-particle realization in $(10,2)$ dimensions, in the purely bosonic context, was given.

The purpose of this paper is to present a supersymmetric extension of the bosonic two-particle model of [8], and to extend further these results to (11,3) dimensions, where a three-particle model arises. We will construct multi-superparticle actions in which the algebras (3) and (4) are realized.

[2] In $(8+n,n)$ dimensions, the full set of generators occuring on the right hand side of $\{Q_\alpha, Q_\beta\}$ are p-form generators with $p = n_0, n_0+4, ..., n+4$, where $n_0 = n$ mod 4. For example, in $(17,9)$ dimensions, there are p-form generators with $p = 1, 5, 9, 13$. However, actions of the type considered here for n-particle systems naturally select the n th rank generator.

In doing so, we will find that the multi-superparticle system has new bosonic local symmetries that generalize the usual reparametrization [8, 13], and new fermionic local symmetries that generalize the usual κ-symmetry of the single superparticle. These symmetries will be shown to exist in presence of super Yang-Mills background as well.

These results can be viewed as preludes to the constructions of higher superbranes in (10,2) and (11,3) dimensions. Since the latter should admit superparticle limits, it is important to develop a better understanding of the superparticle systems in this context.

2 Superparticles in (10,2) Dimensions

2.1 A Model with Two Times

We consider two superparticles which propagate in their respective superspaces with coordinates $X_i^\mu(\tau_1,\tau_2)$ and $\theta_i^\alpha(\tau_1,\tau_2)$, with $i=1,2$, $\mu=0,1,...,11$ and $\alpha=1,...,32$. Working in first order formalism, we also introduce the momentum variables $P_\mu^i(\tau_1,\tau_2)$ [3]. The superalgebra (3) can be realized in terms of supercharges

$$Q_\alpha = Q_{1\alpha} + Q_{2\alpha}\ , \tag{5}$$

with $Q_i(\tau_1,\tau_2)$ defined as [4]

$$Q_{i\alpha} = \partial_{i\alpha} + \tfrac{1}{4}\, \gamma^{\mu\nu}_{\alpha\beta}\, \theta_i^\beta\, P_{1\mu}\, P_{2\nu}\ , \quad i=1,2\ . \tag{6}$$

The spinorial derivative is defined as $\partial_{i\alpha} = \partial/\partial\theta_i^\alpha$, acting from the right. The transformations generated by the supercharges Q_α are

$$\delta_\epsilon X_i^\mu = \tfrac{1}{4}\, \bar\epsilon\gamma^{\mu\nu}(\theta_1+\theta_2)\, \varepsilon_{ij} P_{j\nu}\ , \qquad \delta_\epsilon\theta_i = \epsilon\ , \qquad \delta_\epsilon P_i^\mu = 0\ , \tag{7}$$

where $\bar\epsilon\gamma^{\mu\nu}\theta$ stands for $\epsilon^\alpha\gamma_{\alpha\beta}\theta^\beta$, and ε_{ij} is the constant Levi-Civita symbol with $\varepsilon_{12}=1$. Next, it is convenient to define the line element

$$\Pi_i^\mu = (\partial_1+\partial_2)X_i^\mu - \tfrac{1}{8}\bar\theta_k\gamma^{\mu\nu}(\partial_1+\partial_2)\,\theta_k\, \varepsilon_{ij}P_{j\nu}\ . \tag{8}$$

While this is not supersymmetric by itself, its product with P_μ^i is supersymmetric upto a total derivative term, and therefore it is a convenient building block for an action. The fact that the sum of two times occur in the line element is a consequence of maintaining supersymmetry (in the

[3] In the earlier version of this paper, we considered a restricted dependence of variables on proper times (see (20) and below). We are grateful to I. Bars for stimulating discussions that led us to consider more general proper time dependence.

[4] The spinors are Majorana-Weyl, their indices are chirally projected, the charge conjugation matrix C is suppressed in $(\gamma^{\mu_1\cdots\mu_p}C)_{\alpha\beta}$, which are symmetric for $p=2,3$ in $(10,2)$ dimensions, and for $p=3,4$ in $(11,3)$ dimensions. This symmetry property alternates for p mod 2.

sense just stated) *and* the fact that all field depend on τ_1 and τ_2 (see the end of this section for a discussion of a restricted time dependence, and its consequences).

Introducing the symmetric Lagrange multipliers $A_{ij}(\tau_1,\tau_2)$ which are inert under supersymmetry,

$$\delta_\epsilon A_{ij} = 0 \ , \tag{9}$$

we consider the following action for a two-superparticle system in $(10,2)$ dimensions

$$I = \int d\tau_1 d\tau_2 \left(P_i^\mu \Pi_{i\mu} - \tfrac{1}{2} A_{ij} P_i^\mu P_{j\mu} \right) \ . \tag{10}$$

The action (10) has a number of interesting symmetries. To begin with, it is invariant under the target space rigid supersymmetry transformations (7), up to a total derivative term that has been discarded. Furthermore, it has the local bosonic symmetry

$$\delta_\Lambda A_{ij} = (\partial_1 + \partial_2)\ \Lambda_{ij} \ , \qquad \delta_\Lambda X_i^\mu = \Lambda_{ij} P_j^\mu \ , \qquad \delta_\Lambda P_i^\mu = 0 \ , \qquad \delta_\Lambda \theta = 0 \ , \tag{11}$$

where the transformation parameters have the time dependence $\Lambda_{ij}(\tau_1,\tau_2)$. Here too, a total derivative term, which has the form $(\partial_1 + \partial_2)\left(\frac{1}{2}\Lambda_{ij} P_\mu^i P^{j\mu}\right)$, has been dropped. The diagonal part of these transformations are the usual reparametrizations, combined with a trivial symmetry of the action [14]. The off-diagonal part of the symmetry are gauge symmetries which are the first order form of those which are in the bosonic two-particle model of [8]. Together with the reparametrization symmetries, they allow us to eliminate the correct amount of degrees of freedom to yield 8 bosonic physical degrees of freedom for each particle.

The action (10) has also local fermionic symmetries which generalize the usual κ-symmetries. Let us denote the jth symmetry of the ith particle by $\kappa_{ij}(\tau_1,\tau_2)$. One finds that the action (10) is invariant under the following transformations

$$\begin{aligned}
\delta_\kappa \theta_i &= \gamma^\mu \kappa_{ij} P_{j\mu} \ , \\
\delta_\kappa X_i^\mu &= \tfrac{1}{4}\left(\bar\theta_k \gamma^{\mu\nu} \delta_\kappa \theta_k\right) \varepsilon_{ij} P_{j\nu} \ , \\
\delta_\kappa P_i^\mu &= 0 \ , \\
\delta_\kappa A_{ij} &= \tfrac{1}{2} \bar\kappa_{ki} \gamma^\mu (\partial_1 + \partial_2)\theta_k \ \varepsilon_{\ell j} P_\mu^\ell + (i \leftrightarrow j) \ .
\end{aligned} \tag{12}$$

In showing the κ-symmetry of the action, it is useful to note the lemma

$$P_\mu^i (\delta_\kappa \Pi_i^\mu) = (\partial_1 + \partial_2)\left(\tfrac{1}{8}\bar\theta_k \gamma^{\mu\nu} \delta_\kappa \theta_k \varepsilon_{ij} P_\mu^i P_\nu^j\right) - \tfrac{1}{4}(\delta_\kappa \bar\theta)\gamma^{\mu\nu}(\partial_1 + \partial_2)\theta_k \varepsilon_{ij} P_\mu^i P_\nu^j \ . \tag{13}$$

The diagonal part of the κ_{ij}-transformations are the κ-symmetry transformations that resemble the ones for the usual superparticle. The off-diagonal κ-transformations are their generalizations for a two-superparticle system. Just as the off-diagonal Λ_{ij} transformations are needed to obtain 8 bosonic degrees of freedom for each particle, the off-diagonal κ_{ij} symmetries are needed to obtain

8 fermionic degrees of freedom for each particle. The commutator of two κ-transformations closes on-shell onto the Λ-transformations

$$[\delta_{\kappa_{(1)}}, \delta_{\kappa_{(2)}}] = \delta_{\Lambda_{(12)}} \ , \tag{14}$$

where the composite gauge transformation parameter is

$$\Lambda_{(12)ij} = \tfrac{1}{2}\bar{\kappa}_{(2)ki}\gamma^{\mu\nu}\kappa_{(1)kj}P_{1\mu}P_{2\nu} + (i \leftrightarrow j) \ . \tag{15}$$

It is clear that the remaining part of the algebra is $[\delta_\kappa, \delta_\Lambda] = 0$ and $[\delta_{\Lambda_1}, \delta_{\Lambda_2}] = 0$.

The field equations that follow from the action (10) take the form

$$P_i^\mu P_{j\mu} = 0 \ , \tag{16}$$

$$\gamma^{\mu\nu}P_{1\mu}P_{2\nu}(\partial_1 + \partial_2)\theta_i = 0 \ , \tag{17}$$

$$(\partial_1 + \partial_2)X_i^\mu = \left(A_{ij}\eta^{\mu\nu} + \tfrac{1}{4}\varepsilon_{ij}\bar{\theta}_k\gamma^{\mu\nu}(\partial_1 + \partial_2)\theta_k\right)P_\nu^j \ , \tag{18}$$

$$(\partial_1 + \partial_2)P_{i\mu} = 0 \ . \tag{19}$$

While the derivatives occur only in the combination $(\partial_1 + \partial_2)$, the fields can depend both on τ_+ and τ_- defined by $\tau_\pm \equiv \tau_1 \pm \tau_2$. It is possible, for example, to restrict the proper time dependences as follows

$$X_i^\mu(\tau_i) \ , \qquad P_i^\mu(\tau_i) \ , \qquad \theta_i(\tau_i) \ . \tag{20}$$

The action still is given by (10), with the line element now taking the form

$$\Pi_i^\mu = \partial_i X_i^\mu - \tfrac{1}{4}\bar{\theta}_i\gamma^{\mu\nu}\partial_i \ \theta_i \ \varepsilon_{ij}P_{j\nu} \ . \tag{21}$$

It is understood that the free indices of the left hand side are not not be summed over on the right hand side. The bosonic and fermionic symmetries discussed earlier retain their forms. In particular, the global supersymmetry transformations (7) remain the same, and one can express the bosonic symmetries (11) as

$$\delta_\Lambda A_{ij} = \partial_i\Lambda_{ij} + \partial_j\Lambda_{ji} - \delta_{ij}\partial_i\Lambda_{ij} \ , \qquad \delta_\Lambda X_i^\mu = \Lambda_{ij}P_j^\mu \ , \qquad \delta_\Lambda P_i^\mu = 0 \ , \qquad \delta_\Lambda\theta = 0 \ , \tag{22}$$

where the parameters Λ_{ij} depend on τ_1 and τ_2. Restricting the proper time dependence of the κ-symmetry parameter as $\kappa_{ij}(\tau_i)$, the fermionic symmetries (12) can be simplified to take the form

$$\begin{aligned}
\delta\theta_i &= \gamma^\mu\kappa_{ij}P_{j\mu} \ , \\
\delta X_i^\mu &= \tfrac{1}{4}\left(\bar{\theta}_i\gamma^{\mu\nu}\delta\theta_i\right)\varepsilon_{ij}P_{j\nu} \ , \\
\delta P_i^\mu &= 0 \ , \\
\delta A_{ij} &= \tfrac{1}{2}\left(\bar{\kappa}_{kj}\gamma^\mu\partial_k\theta_k + \bar{\kappa}_{ij}\gamma^\mu\partial_i\theta_i\right)\varepsilon_{ki}P_{k\mu} + (i \leftrightarrow j) \ ,
\end{aligned} \tag{23}$$

which close on-shell as in (14), and the composite parameter now takes the form

$$\Lambda_{(12)ij} = \tfrac{1}{2}\bar{\kappa}_{(2)ii}\gamma^{\mu\nu}\kappa_{(1)ij}P_{1\mu}P_{2\nu} - (1 \leftrightarrow 2) \ . \tag{24}$$

Similarly, the field equations become

$$P_i^\mu P_{j\mu} = 0 \ , \qquad \partial_i P_{i\mu} = 0 \ , \qquad \gamma^{\mu\nu} P_{1\mu} P_{2\nu} \partial_i \theta_i = 0 \ , \tag{25}$$

$$\partial_i X_i^\mu = \left(A_{ij} \eta^{\mu\nu} + \tfrac{1}{2} \varepsilon_{ij} \bar{\theta}_i \gamma^{\mu\nu} \partial_i \theta_i \right) P_{j\nu} \ . \tag{26}$$

Note that the κ-symmetry transformation of, say X_1^μ, maps a function of τ_1 to a function of τ_1 *and* τ_2. While this may seem somewhat unusual, it does not present any inconsistency, and in particular, there is no need to take the momenta to be constants. The important point to bear in mind is that the symmetry transfomations close and that they are consistently embedded in a larger set of transformations that map functions of (τ_1, τ_2) to each other.

2.2 Putting the Second Particle On-Shell

In order to obtain an action for the first particle propagating in the background of the second particle, we will follow the following procedure. Recall that $\tau_\pm \equiv \tau_1 \pm \tau_2$. Let us also define $\theta_\pm \equiv \theta_1 \pm \theta_2$. We use the X_2, A_{22}, θ_- equations of motion in the action, thereby putting the second particle on-shell. However, the remaining fields still have τ_- dependence. In analogy with Kaluza-Klein reduction, we then set $\partial_- = 0$, and integrating the action over τ_- to obtain:

$$I = \int d\tau \left[P_\mu \left(\Pi^\mu - A n^\mu \right) - \tfrac{1}{2} e P^\mu P_\mu \right] \ . \tag{27}$$

where the label 1 has been suppressed throughout, the τ_- interval is normalized to 1 and

$$\begin{aligned} & P_2^\mu \equiv n^\mu \ , \\ & A_{11} \equiv e \ , \quad A_{12} \equiv A \ , \\ & \Pi^\mu = \partial_\tau X^\mu - \tfrac{1}{4} \bar{\theta} \gamma^{\mu\nu} \partial_\tau \theta \ n_\nu \ , \end{aligned} \tag{28}$$

where we have defined $\tau_+ \equiv \tau$ and $\theta_+ \equiv \theta/\sqrt{2}$. The vector n^μ is constant and null as a consequence of X_2 and A_{22} equation of motion, and the fact that we have set $\partial_- = 0$.

The action (27) is invariant under the local bosonic transformations

$$\delta e = \partial_\tau \xi \ , \qquad \delta A = \partial_\tau \Lambda \ , \qquad \delta X^\mu = \xi P^\mu + \Lambda n^\mu \ , \qquad \delta P^\mu = 0 \ , \qquad \delta\theta = 0 \ , \tag{29}$$

obtained from (11) by setting $\partial_- = 0$ and using the notation $\xi \equiv \Lambda_{11}$ and $\Lambda \equiv \Lambda_{12}$. The action (27) is also invariant under the global supersymmetry transformations (7)

$$\delta_\epsilon X^\mu = \tfrac{1}{4} \ \bar{\epsilon} \gamma^{\mu\nu} \theta \ n_\nu \ , \qquad \delta_\epsilon \theta = \epsilon \ , \qquad \delta_\epsilon P^\mu = 0 \ , \qquad \delta_\epsilon e = 0 \ , \qquad \delta_\epsilon A = 0 \ , \tag{30}$$

(we have rescaled $\epsilon \to \epsilon/\sqrt{2}$ for convenience) and invariant under the local fermionic κ and η transformations

$$\delta\theta \ = \ \gamma^\mu P_\mu \kappa + \gamma^\mu n_\mu \eta \ ,$$

$$
\begin{aligned}
\delta X^\mu &= \tfrac{1}{4}\bar{\theta}\gamma^{\mu\nu}n_\nu\left(\delta_\kappa\theta+\delta_\eta\theta\right) ,\\
\delta P^\mu &= 0 ,\\
\delta e &= -\bar{\kappa}\gamma^\mu\partial_\tau\theta\ n_\mu ,\\
\delta A &= \tfrac{1}{2}\bar{\kappa}\gamma^\mu P_\mu\partial_\tau\theta-\tfrac{1}{2}\bar{\eta}\gamma^\mu n_\mu\partial_\tau\theta ,
\end{aligned}
\tag{31}
$$

obtained from (12) by setting $\partial_- = 0$, using the field equations of the second particle, setting $\kappa_{-i} = 0$, and using the notation $\kappa_{+1} \equiv \sqrt{2}\kappa$ and $\kappa_{+2} \equiv \sqrt{2}\eta$. The parameter κ_{-i} has been set equal to zero, as it is associated with the transformations of θ_- that has been put on-shell, and which has consequently dropped out in the action.

An alternative way to arrive at the same results is to start from the restricted model described at the end of Sec. (2.1), again by putting the second particle on-shell, and this time integrating over τ_2.

2.3 Introducing Super Yang-Mills Background

The coupling of Yang-Mills background is best described in the second order formalism. Elimination of P^μ in (44) gives

$$
I_0 = \tfrac{1}{2}\int d\tau\ e^{-1}\Pi^\mu\left(\Pi_\mu - An_\mu\right) . \tag{32}
$$

The bosonic and fermionic symmetries of this action can be read off from (46) and (48) by making the substitution $P^\mu \to e^{-1}(\Pi^\mu - An^\mu)$. To couple super Yang-Mills background to this system, we introduce the fermionic variables $\psi^r,\ r = 1, ..., 32$, assuming that the gauge group is $SO(32)$. The Yang-Mills coupling can then be introduced as

$$
I_1 = \int d\tau\ \psi^r\partial_\tau\psi^s\partial_\tau Z^M A^{rs}_M, \tag{33}
$$

where Z^M are the coordinates of the $(10, 2|32)$ superspace, and A^{rs}_M is a vector superfield in that superspace.

The torsion super two-form $T^A = dE^A$ can be read from the superalgebra (3):

$$
T^c = e^\alpha\wedge e^\beta\,(\gamma^{cd})_{\alpha\beta}\,n_d ,\qquad T^\alpha = 0 , \tag{34}
$$

where the basis one-forms defined as $e^A = dZ^M E_M{}^A$ satisfy

$$
de^c = e^\alpha\wedge e^\beta\ (\gamma^{cd})_{\alpha\beta}\ n_d ,\qquad de^\alpha = 0 , \tag{35}
$$

and $a, b, c, ...$ are the (10,2) dimensional tangent space indices.

Using these equations, a fairly standard calculation [16, 17] shows that the total action $I = I_0+I_1$ is invariant under the fermionic gauge transformations provided that the Yang-Mills super two-form is given by [5]

$$
F = e^\alpha\wedge e^b\left[\,n_b\chi_\alpha - 2(\gamma_b\lambda)_\alpha\,\right] + \tfrac{1}{2}e^a\wedge e^b\ F_{ba} , \tag{36}
$$

where we have introduced the chiral spinor superfield χ_α and the anti-chiral spinor superfield λ, and that the transformation rules for e and A pick up the extra contributions

$$\begin{aligned}
\delta_{\rm extra} e &= -4e\psi^r\partial_\tau\psi^s(\bar\kappa\lambda_{rs}) \ , \\
\delta_{\rm extra} A &= \psi^r\partial_\tau\psi^s\bar\kappa\left(2\lambda_{rs}+(\Pi^a-An^a)\right)\gamma_a\chi_{rs}+e\psi^r\partial_\tau\psi^s\bar\eta\left(4\lambda_{rs}-\gamma_a n_a\chi_{rs}\right) \ , \\
\delta_{\rm extra}\psi^r &= -\delta\theta^\alpha\ A_\alpha^{rs}\psi^s \ .
\end{aligned} \tag{37}$$

In addition, the spinor λ must satisfy the condition $n^a\gamma_a\lambda=0$. One can show that F given in (36) satisfies the Bianchi identity $DF=0$, and the constraints on F implied by (36) lead to the field equations of the super Yang-Mills system in (10,2) dimensions [5].

3 Superparticles in (11,3) Dimensions

3.1 A Model with Three Times

In this section, we consider three superparticles propagating in $(11,3)$ dimensional spacetime, and take their superspace coordinates to be $X_i^\mu(\vec\tau),\theta_i^\alpha(\vec\tau)$ and momenta $P_i^\mu(\vec\tau)$ $(i=1,2,3)$, where $\vec\tau=(\tau_1,\tau_2,\tau_3)$. Following the same reasoning as in Sec. (2.1), we consider the following action

$$I=\int d\tau_1 d\tau_2 d\tau_3\ \left(P_i^\mu\Pi_{i\mu}-\tfrac12 A_{ij}P_i^\mu P_{j\mu}\right) \ , \tag{38}$$

where

$$\Pi_i^\mu=(\partial_1+\partial_2+\partial_3)X_i^\mu-\tfrac{1}{36}\bar\theta_k\gamma^{\mu\nu\rho}(\partial_1+\partial_2+\partial_3)\theta_k\ \varepsilon_{ijk}P_{j\nu}P_{k\rho} \ . \tag{39}$$

The action is invariant under the local bosonic transformations

$$\delta_\Lambda A_{ij}=(\partial_1+\partial_2+\partial_3)\ \Lambda_{ij} \ , \qquad \delta_\Lambda X_i^\mu=\Lambda_{ij}P_j^\mu \ , \qquad \delta_\Lambda P_i^\mu=0 \ , \qquad \delta_\Lambda\theta=0 \ . \tag{40}$$

The action is also invariant (modulo discarded total derivative terms) under the global supersymmetry transformations

$$\delta_\epsilon X_i^\mu=\tfrac{1}{12}\ \bar\epsilon\gamma^{\mu\nu\rho}(\theta_1+\theta_2+\theta_3)\ \varepsilon_{ijk}P_{j\nu}P_{k\rho} \ , \qquad \delta_\epsilon\theta_i=\epsilon \ , \qquad \delta_\epsilon P_i^\mu=0 \ , \qquad \delta_\epsilon A_{ij}=0 \ , \tag{41}$$

and local fermionic transformations

$$\begin{aligned}
\delta_\kappa\theta_i &= \gamma^\mu\kappa_{ij}P_{j\mu} \ , \\
\delta_\kappa X_i^\mu &= \tfrac{1}{12}\left(\bar\theta_k\gamma^{\mu\nu\rho}\delta\theta_k\right)\varepsilon_{imn}P_\nu^m P_\rho^n \ , \\
\delta_\kappa P_i^\mu &= 0 \ , \\
\delta_\kappa A_{ij} &= \tfrac16\bar\kappa_{ki}\gamma^{\mu\nu}(\partial_1+\partial_2+\partial_3)\theta_k\varepsilon_{jmn}P_\mu^m P_\nu^n+(i\leftrightarrow j) \ .
\end{aligned} \tag{42}$$

The algebra closes on-shell as in (15), with the composite gauge parameter now given by

$$\Lambda_{(12)ij}=\tfrac13\bar\kappa_{(2)ki}\gamma^{\mu\nu\rho}\kappa_{(1)kj}P_{1\mu}P_{2\nu}P_{3\rho}+(i\leftrightarrow j) \ . \tag{43}$$

The remaining part of the algebra is $[\delta_\kappa, \delta_\Lambda] = 0$ and $[\delta_{\Lambda_1}, \delta_{\Lambda_2}] = 0$. The equations of motion are similar to (16)-(19). In fact, all the formulae of this section are very similar to those for two-superparticles given in Sec. (2.1) and their n superparticle extension is straightforward.

3.2 Putting the Second and Third Particles On-Shell

To obtain an action which describes the propagation of the first particle in the background of the other two particles, we follow the steps described in Sec. (2.2). Let $\tau \equiv \tau_1 + \tau_2 + \tau_3$, and $\theta \equiv \theta_1 + \theta_2 + \theta_3$, and denote the orthogonal combinations by $\tau_\pm$ and $\theta_\pm$. Using the equations of motion for $\theta_\pm, X_i, A_{ii}$ $(i = 2, 3)$ and restricting the proper time dependence of fields by setting $\partial_\pm = 0$, we obtain the action

$$I = \int d\tau \left[-\tfrac{1}{2} e P^\mu P_\mu + P_\mu \left(\Pi^\mu - A n^\mu - B m^\mu \right) \right] \ , \tag{44}$$

where $A \equiv A_{12}$ and $B \equiv A_{13}$ and

$$\Pi^\mu = \partial_\tau X^\mu - \tfrac{1}{6} \bar\theta \gamma^{\mu\nu\rho} \partial_\tau \theta \ n_\nu m_\rho \ . \tag{45}$$

The vectors n^μ and m_μ are mutually orthogonal constant null vectors, as a consequence of A_{ii} and X_i^μ equations of motion for $i = 2, 3$ resulting from the original action (38), and having set $\partial_\pm = 0$. The action (44) is invariant under the bosonic transformations

$$\delta e = \partial_\tau \xi \ , \qquad \delta A = \partial_\tau \Lambda \ , \qquad \delta B = \partial_\tau \Sigma \ , \qquad \delta X^\mu = \xi P^\mu + \Lambda n^\mu + \Sigma m^\mu \ , \tag{46}$$

where $\xi \equiv \Lambda_{11}$, $\Lambda \equiv \Lambda_{12}$ and $\Sigma \equiv \Lambda_{13}$. The action is also invariant under the global supersymmetry transformations

$$\delta_\epsilon X^\mu = \tfrac{1}{12} \ \bar\epsilon \gamma^{\mu\nu\rho} \theta \ n_\nu m_\rho \ , \qquad \delta_\epsilon \theta = \epsilon \ , \qquad \delta_\epsilon P^\mu = 0 \ , \qquad \delta_e A = 0 \ , \qquad \delta_e B = 0 \ , \tag{47}$$

and local fermionic κ, η and ω transformations

$$\begin{aligned}
\delta\theta &= \gamma^\mu P_\mu \kappa + \gamma^\mu n_\mu \eta + \gamma^\mu m_\mu \omega \ , \\
\delta X^\mu &= \tfrac{1}{6} \bar\theta \gamma^{\mu\nu\rho} n_\nu m_\rho \left(\delta_\kappa \theta + \delta_\eta \theta + \delta_\omega \theta \right) \ , \\
\delta P^\mu &= 0 \ , \\
\delta e &= -\tfrac{2}{3} \bar\kappa \gamma^{\mu\nu} \partial_\tau \theta n_\mu m_\nu \ , \\
\delta A &= -\tfrac{1}{3} \bar\kappa \gamma^{\mu\nu} \partial_\tau \theta m_\mu P_\nu - \tfrac{1}{3} \bar\eta \gamma^{\mu\nu} \partial_\tau \theta n_\mu m_\nu \ , \\
\delta B &= -\tfrac{1}{3} \bar\kappa \gamma^{\mu\nu} \partial_\tau \theta P_\mu n_\nu - \tfrac{1}{3} \bar\kappa \gamma^{\mu\nu} \partial_\tau \theta n_\mu m_\nu - \tfrac{1}{3} \bar\omega \gamma^{\mu\nu} \partial_\tau \theta n_\mu m_\nu \ ,
\end{aligned} \tag{48}$$

where κ, η, ω are equivalent to $\kappa_{+i} (i = 1, 2, 3)$ upto a constant rescaling, and the irrelevant parameters $\kappa_{\pm i}$ have been set equal to zero.

4 Conclusions

We have presented simple action formulae for two- and three-superparticle systems in $(10,2)$ and $(11,3)$ dimensions, respectively. The symmetries of the action exhibit interesting generalizations of reparametrization and κ-symmetries. An action similar to (32)-(33) can easily be constructed for the $(11,3)$ dimensional superparticle and it implies the $(11,3)$ dimensional super Yang-Mills equations [15]. We also expect that the action (32)-(33) can be generalized to obtain a heterotic string action and possibly other superbrane actions.

The n particle models constructed here ($n = 2,3$) make use of n fermionic coordinates θ_i ($i = 1,...,n$). The fact that they all transform by the same constant parameter ε suggests that we can identify them: $\theta_i = \theta$. It is also natural from the group theoretical point of view to associate the coordinates X_i^μ, θ_α with the generators P_i^μ, Q_α. However, it is not necessary to do so, since there are sufficiently many local fermionic symmetries to give 8 physical fermionic degrees of freedom for each θ_i (see (12)). Thus, it does not seem to be crucial to have one or many fermionic variables.

Another, and possibly more significant, feature of the models constructed here is that they involve multi-times, in the sense that fields depend on τ_i ($i = 1,...,n$) over which the action is integrated over. The derivatives occuring in the action come out to be the sum of these times, which indicates that any (pseudo) rotational symmetry among them is lost. One might therefore be tempted to declare the fields to depend on the single time $\tau = \tau_1 + \cdots \tau_n$. Perhaps this is the sensible thing to do, however, we have kept the multi-time dependence here, partically motivated by the fact that our results may give a clue for the construction of an action that involves an (n,n) dimensional worldvolume. If an action can indeed be constructed for (n,n) brane, i.e. brane with an (n,n) dimensional worldvolume, one may envisage a 'particle limit' in which the spatial dependence is set equal to zero, yielding an $(n,0)$ dimensional worldvolume. If the worldvolume diffeomorphisms are to be maintained, then we may need to consider a term of the form $P_\mu^{ia}\partial_a X_i^\mu$ in the action, where $a = 1,...,n$ labels the $(n,0)$ worldvolume coordinates. It is not clear, however, how supersymmetry and κ-symmetry can be achieved in this setting.

Notwithstanding these open problems, one may proceed to view the coordinates (τ_1,σ_1) and (τ_2,σ_2) as forming a $(2,2)$ dimensional worldvolume embedded in $(10,2)$ dimensions, in the case of a two-superstring system. Similarly, a three-superstring system would form a $(3,3)$ dimensional worldvolume embedded in $(11,3)$ dimensions.

It is of considerable interest to construct a string theory in $(3,3)$ dimensions which would provide a worldvolume for a $(3,3)$ superbrane propagating in $(11,3)$ dimensions [10], thereby extending the construction of [2, 3] a step further [15]. Indeed, a string theory in (n,n) (target space) dimensions has been recently constructed [15]. This theory is based on an $N = 2$ superconformal algebra for the right-movers in (n,n) dimensions, and an $N = 1$ superconformal algebra for the left-movers in (8+n,n) dimensions. The realizations of these algebras contain suitable number of constant null vectors, which arise in the expected manner in the algebra of supercharge vertex operators [9]. Furthermore, the massless states are expected to assemble themselves into super Yang-Mills multiplet in $(8+n,n)$ dimensions. Much remains to be done towards a better understanding of this theory, and its implications for the target space field equations that may exhibit interesting geometrical structures that generalize the self-dual Yang-Mills and gravity equations in $(2,2)$ dimensions [19].

Further studies of supersymmetry in $D>11$ may also be motivated by the fact that they contain both the type IIA and IIB supersymmetries of ten dimensional strings [12, 15]. Therefore, it would be interesting to find a brane-theoretic realization of the $D>11$ symmetries that would provide a unified framework for the description of all superstrings in $(9,1)$ dimensions.

Acknowledments

We dedicate this article to the memory of D.V. Volkov who is one of the fathers of supersymmetry and whose original work on the theory of superparticles and superbranes has been very inspirational to us. We are grateful to I. Bars, C.S. Chu and R. Percacci for many helpful discussions. We also thank Z. Khviengia for a useful discussion concerning the structure of the three-particle action and V. Dobrev for pointing out reference [18] to us. This work was suppoted in part by the National Science Foundation, under grant PHY-9722090.

The following Note Added expands the Note Added that appeared in the previous version of this paper, in order to explain in more detail the revisions that were made in the first version of the paper and its relation with the papers of Bars and Deliduman [21, 22]. This version of the Note Added is to appear as an Addendum in Phys. Lett. B.

Note Added

In the original version of this paper, we proposed the action (10), with the line element as given in (21), and time dependences of the variables as in (20). We gave the κ-symmetry transformations (23), the supersymmetry transformations (7), the bosonic symmetry transformations (22) (with some minor errors corrected here) and the symmetry algebra (14), again with the restricted time dependences (21) understood. The original version also contained the correct results for the description of one superparticle in the background of one or two other superparticles with constant momenta, as well as a super Yang-Mills background.

Subsequently, Ref. [21] appeared in which an action for a superparticle in the backcground of a second superparticle with constant momentum was also constructed. The single time formalism was introduced in [21]. Allowing gauge transformations that depend on the equations of motion, the results of [21] agree with ours.

Eventually, in Ref. [22], these models were improved considerably and an action was proposed for multi-superparticles in which the variables depend on single time τ, and the momenta are not taken to be constants.

After the appearance of [22], and motivated by discussions with I. Bars about the property of the symmetry transformations of our original model, in which a function of one time is mapped into a function of another time, we revisited the original version of our paper and considered a multi-time dependence for all the variables, to see if we could relax this property. Performing the Noether procedure, we found that the line elements had to be defined as in (8), and the symmetries as in (11) and (12). However, observing that: a) only the derivative with respect to the total time occurs in the action, and b) the index on the fermionic coordinates, and the first index on the κ-symmetry parameter can be viewed as an extended supersymmetry index (denoted by A in [22]), it follows that the model thus obtained is in effect that of Bars and Deliduman [22].

It should be noted that the multi-time approach: a) enables one to see the connection between the the single time model of Bars and Deliduman [22] and our original model with the restricted time dependence of variables and b) shows that the symmetry transformations that map a function of one time into a function of another time can consistently be embedded into a larger set of transformations that map functions of both times into each other (see below (26) for further comments about this, and the third paragraph in the conclusions for a further discussion of the multi-time approach).

We also note that, focusing on the case of simple supersymmetry, while the κ-transformation rules of Ref. [22] contained one κ-symmetry parameter (though the existence of n fermionic first class constraints was observed), we found the explicit form of the κ-symmetry transformations that involve n such parameters. A detailed account of the relation between the parameters, and the gauge transformations proportional to the field equations that explain the different forms of the transformation rules is given in [23].

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